Tale of the Oracle

Mariam Kiran
Department of Computer Science
University of Sheffield,
Regent Court, 211 Portobello Street,
S14DP Sheffield, United Kingdom
acp05mk@shef.ac.uk

24 August 2006

Abstract

The aim of this paper is to analyze oracular power drawing a few conclusions on their natures and possible implementations in other models. The secrets of the power of the oracle lies in the inner working of the oracle itself. This paper attempts to discuss the powers of these oracles (acting as black boxes) and how their hierarchies can be used to extend powers *containing* much stronger powerful models. The stability of such hierarchies has also been analyzed to reveal possible implementations of the oracle extending discussions to how it can be used to simulate more complex models like the mind.

1 Introduction

Coining the term *oracle*, Turing [Tur39] refers to the examples at Delphi describing the oracle machines as prophetic entities to provide answers to the Turing machine. Addressing it as 'not a machine', Turing depicted the first image of hypercomputational ability to solve functions which the Turing machine alone was unable to. The nature of these oracles has been subject to discussions since their presentation in 1939. Scholars like Cleland [Cle00, Cle02] and Dana Scott [Dav04] have argued about what are oracles and their non-recursive powers. Cleland [Cle02] goes further in her discussions by saying that 'it doesn't matter *how* an oracle does what it does; all that matters is *what* it does'. This paper however, attempts to analyze exactly how these oracles are able to do their tasks assisting the Turing machines and extending their use due to their increasing powers in oracular hierarchy.

2 Simply Magical?

The oracles are more than just 'bedfellows' to the Turing machines, as perceived by Bringsjord and Zenzen [BZ02]. Turing had presented them as black boxes which the Turing machine would use to gain answers to undecidable problems, which it alone would not have. The machine would just ask the oracle the question and it would be provided with an answer straightaway. Neuman [Neu55] claimed that the oracle is similar to a human mathematician 'having an idea'. This claim was later contradicted by Hodges [Hod02] which he justified that a human required time to think before providing an answer whereas the oracle responds immediately. However, describing oracles as mathematical tools, Hodges related the oracle to an 'intuition involved in seeing the truth in Gödel statements'. Conversely, using the Church-Turing Thesis [Cop95] definitions, if the human with pencil and paper was allowed intelligence it would provide the oracle to the Turing machine.

Copeland [Cop02] has described the oracle as a table using an exhaustive specification providing answers, to the machine, of what to do when such a symbol is found. This definition could be extended to show the possible implementation of the oracle (Definition 2.1).

Definition 2.1 *If a Turing machine was provided with an infinite lookup table which would give y output when x input is received, then the lookup table would act as an oracle.*

Oracles have been related to human intuition by various scholars [Neu55, Hod02]. Even Turing [Tur50] argued that human intelligence or intuition grows from learning when introducing the Turing Test as a manner of measuring intelligence. We can argue that human intelligence is dependent on the environment signals its mind is processing from childhood to adult. Figure 1 represents a depiction of this scenerio[1].

Definition 2.2 *If a machine M was processing z inputs from the environment, continually updating its infinite lookup table, these inputs would influence the outputs y it produced on x inputs.*

3 Limited Hierarchy

If an oracle was fed as input onto the tape of a Turing machine which was using another oracle, the modified machine would be able to process functions by both oracles. Conversely, the hierarchy in oracular machines have showed how their power increases with increasing the the number of oracles written onto the tape. This argument could be used to represent the oracle machines as the most powerful hypercomputational model[2]. The secret of the power lies in the inner workings of the oracle. Each oracle could be coded

[1] Copeland's [CS99] coupled Turing machine suggests possible implementation of how the Turing machine would be taking continuous inputs during execution similar to the machine depicted in Figure 1.

[2] These results have been discussed in [Kir06].

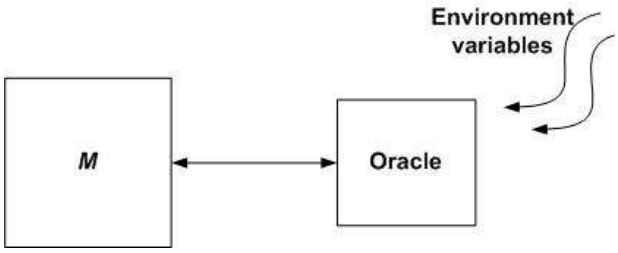

Figure 1: Oracle simulating the mind

to another oracle enhancing the other oracle's to a higher degree of power. The modified machine can now execute its own functions and the functions which the prior machine could solve. Increasing functionality of the machine increases the power of these machines.

Conversely, the argument of the power of the oracles gives the conclusion that every function in the universe can be processed if the given oracle has the information to process it. Therefore oracles would be able to process controversial models of the mind or the universe completely, if allowed to do so. However, providing such unlimited power to a model is questionable. If there existed such a super oracle which would be able to process every function of any other oracle, there could be no proof if that super oracle would not have a oracle more powerful than itself. There would always be a possibility of higher powers to be attained making the argument of power unstable. Consequently, a super oracle would require an uncountable number of oracles written on its tape, which would also be of uncountable number of squares. This would again violate the hypercomputational claim.

4 Conclusion

Section 3 gives a description of the confusions on the power of hierarchies in oracles. The hierarchies could be a manner in which power cold be contained on to every next level in the oracular power. Conversely this allows the oracle to be able to process anything, even a complex object like the mind[3]. However, the undetermined super oracle raises the question of whether the argument of hierarchies in oracles is substantial or not.

The possible implementation of the mind using an oracle is described in Section 2. It represents a manner in which intelligence could be simulated, based on the environment the machine exists in. Taking regular inputs from the environment, which would be non-recursive, the machine might be able to process functions which are different from functions being processed by a different machines existing in separate environment. This scenario justifies the different characteristics in humans and their dependencies on the environment they live in.

The oracles are truly a very complex structure to simulate non recursive functions.

[3]Refer to Section 2.

Their implementations are very similar to an infinite lookup table which would continue adding more information into it as it is observed. Such a model would be similar in structure to the infinite memory Turing machine. Its continually updating table would allow the oracle to hold as many answers, allowing if asked to refer to this table and provide the answer. However, the conditions which the machine will use to check for referring to the answer from the table can not be generalized. These depend on the functions the oracle is responsible for performing and the parameters it is using, again restricting its domain of activity. Consequently, the oracle does show promises in hypercomputational abilities to process more complex structures which other models might not be able to, however future research in this direction is likely to reveal additional properties of oracles and may contribute to our understanding of their complex natures and possibly undefined hierarchies.

Acknowledgement

The results drawn in this paper were attained during my MSc dissertation [Kir06]. I would like to express deep gratitude to my supervisor Dr. Mike Stannett whose support and guidance allowed successful completion of the project and the results in this paper.

References

[BZ02] Selmer Bringsjord and Michael Zenzen. Towards a formal philosophy of Hypercomputation. In *Minds and Machines*, volume 12, pages 241–258. Kluwer Academic Publishers, Netherlands, 2002.

[Cle00] Carol Cleland. Effective procedures and causal processes. *Hypercomputation Workshop*, May 2000. London, England.

[Cle02] Carol E. Cleland. On effective procedures. In *Minds and Machines*, volume 12, pages 159–179, Netherlands, 2002. Kluwer Academic Publishers.

[Cop95] Jack Copeland. The Church-Turing Thesis. In *Stanford Encyclopedia of Philosophy*. Berlin, 1995. <Online:`http://plato.stanford.edu/entries/church-turing/`>.

[Cop02] Jack B. Copeland. Accelerating turing machines. In *Minds and Machines*, volume 12, pages 281–301, Netherlands, 2002. Kluwer Academic Publishers.

[CS99] Jack B. Copeland and Richard Sylvan. Beyond the universal Turing machine. *Australian Journal of Philosophy*, 77:46–66, 1999.

[Dav04] Martin Davis. The Myth of Hypercomputation. pages 195–211. 2004. Appeared in [Teu04].

[Hod02] Andrew Hodges. Alan Turing. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Summer 2002. <Online: `http://plato.stanford.edu/archives/sum2002/entries/turing/`>.

[Kir06] Mariam Kiran. Comparative analysis of hypercomputational systems. Master's thesis, Department of Computer Science, University of Sheffield, Sheffield, UK, August 2006.

[Neu55] M. H. A. Neuman. Alan Mathison Turing. *Biographical memoirs of the Royal Society*, pages 253–263, 1955.

[Teu04] Christof Teuscher, editor. *Alan Turing: Life and Legacy of a Great Thinker*, Berlin Heidelberg New York, 2004. Springer-Verlag.

[Tur39] A.M. Turing. Systems of Logic Based on the Ordinals. *Proceedings of the London Mathematical Society*, (45):161–228, 1939. This was Turing's Ph.D. thesis, Princeton University (1938). <Online:`http://www.turing.org.uk/philosophy/ex3.html`>.

[Tur50] Alan M. Turing. Computing machinery and intelligence. *Mind*, 59:433–460, 1950.