An Example of Hypercomputation in Algebra and Logic

Benjamin Wells
University of San Francisco

During the workshop, I intend to discuss recent research in hypercomputation related to these four papers, each followed by its abstract:

"Is There a Nonrecursive Decidable Equational Theory?" *Minds and Machines* **12** (2002), pp. 303–326.

Abstract
The Church-Turing Thesis (CTT) is often paraphrased as "every computable function is computable by means of a Turing machine." The author has constructed a family of equational theories that are not Turing-decidable, that is, given one of the theories, no Turing machine can recognize whether an arbitrary equation is in the theory or not. But the theory is called pseudorecursive because it has the additional property that when attention is limited to equations with a bounded number of variables, one obtains, for each number of variables, a fragment of the theory that is indeed Turing-decidable. In a 1982 conversation, Alfred Tarski announced that he believed the theory to be decidable, despite this contradicting CTT. The article gives the background for this proclamation, considers alternate interpretations, and sets the stage for further research. © 2002 Springer Netherlands.

"Hypercomputation by Definition," *Theoretical Computer Science* **317** (2004), pp. 191-207; published online at Elsevier ScienceDirect.

Abstract
Hypercomputation refers to computation surpassing the Turing model, not just exceeding the von Neumann architecture. Algebraic constructions yield a finitely based pseudo-recursive equational theory (Internat. J. Algebra Comput. 6 (1996) 457–510). It is not recursive, although for each given number *n*, its equations in *n* variables form a recursive set. Hypercomputation is therefore required for an algorithmic answer to the membership problem of such a theory. Yet Alfred Tarski declared these theories to be decidable. The dilemma of a decidable but not recursive set presents an impasse to standard computability theory. One way to break the impasse is to predicate that the theory is computable—in other words, hypercomputation by definition. © 2003 Elsevier B.V.

"Applying, Abstracting, Extending, and Specializing Pseudorecursiveness," *Annals of Pure and Applied Logic* **126** (2004), pp. 225–254; published online at Elsevier ScienceDirect.

Abstract
Pseudorecursive varieties (Inter. J. Algebra Comput. 6 (1996) 457) exhibit a lack of recursive uniformity, expressing the failure of universal and existential quantifiers to reverse. Several examples are given of personal encounters with infeasible or errant quantifier reversal. Results strengthening and applying pseudorecursiveness are followed by the study of a property of spectra that is not uniform. These foreshadow an abstraction of this notion and its integration with the algebraic and computational studies—steps that may eventually help explicate Tarski's claim that recursively enumerable, nonrecursive but pseudorecursive equational theories are nonetheless decidable.

"Abstracting and Generalizing Pseudorecursiveness," to be submitted for a special volume on the life and work of Andrzej Mostowski. (11 pp.)

Abstract
Pseudorecursive varieties (Inter. J. Algebra Comput. 6 (1996) 457) strongly express a lack of recursive uniformity related to equational logic. This article suggests an abstraction of the notion and its integration with algebraic and computational studies—steps that may support or at least explicate Tarski's claim that the nonrecursive pseudorecursive equational theories are nonetheless decidable.

My main areas of teaching and research involvement lie in mathematics (logic and algebra) and computer science (theory and computer graphics). Besides the papers listed above, my work in hypercomputation includes refereeing an article for *Theoretical Computer Science*, reviewing a chapter for another author, and reviewing a book for an international publisher. Martin Davis and I periodically exchange news and opinions on hyper-computation at UC Berkeley logic talks.